THE LOGICAL DIFFERENCE BETWEEN KNOWLEDGE AND JUSTIFIED BELIEF

1. LOGICAL BACKGROUND Historically, the study of logic has had both an epistemic and an ontological dimension. From Aristotle until the mid-nineteenth century, the focus of logic was primarily epistemic. Logic dealt with argument, deduction, and proof. It provided the resources to distinguish correct from incorrect arguments, and was thought to be of some help in constructing correct arguments, and in using reason to extend our knowledge. From the time of Boole and Frege, logic turned in an ontological direction. Logic became concerned with devising languages that adequately represent the world and its contents, with exploring the expressive power of sentences and sets of sentences, and with establishing "laws" that are concerned with how things are rather than with how to reason about the things that are. Frege seems to have disdained epistemology, probably because he failed to properly distinguish epistemology from psychology.

Although logicians and teachers of logic, especially teachers of elementary courses in logic, are concerned with the epistemic as well as with the ontological, it is difficult to accommodate epistemic concerns in contemporary logical theory. Deductive systems are frequently not well suited to exploring arguments; their task is to codify semantically distinguished sentences or statements, or abstract arguments constituted by a set of premisses together with a conclusion. The semantic, which is basically ontological, "calls the shots" in logic.

I think it is more appropriate to give "equal time" to both the ontological and the epistemic. Neither should be pursued at the expense of the other. Illocutionary logic, or the logic of speech acts, is well suited to achieve this goal, accommodating both the ontic and the epistemic. As I conceive it, this study is broader than standard logic, and includes standard logic as a proper part.

I take the fundamental "linguistic reality" to be constituted by speech acts, or *language acts*. A speech act is a meaningful act performed by using an expression. A speech act or language act can be performed by speaking, writing, or thinking with words. On this understanding, a speech act/language act doesn't require anyone to speak out loud, and it doesn't require that the speech actor have an audience. Language acts are the primary bearers of such semantic features as meaning and truth. Written and spoken expressions are the bearers of syntactic features, and can themselves be regarded as syntactic objects. Although it is language acts that are meaningful, various expressions are conventionally used to perform acts with particular meanings; the meanings commonly assigned to expressions are the meanings of acts they are conventionally used to perform.

A logical treatment of language acts gives some prominence to *sentential* acts, those language acts performed with complete sentences. Among sentential acts, *propositional* acts are of particular interest–these are the sentential acts that are true or false. Because it is a little cumbersome to say 'propositional act' repeatedly, I also call these acts *statements*. This is a stipulated meaning for 'statement,' and is different from its more common use to mean something like 'assertion.' Sentential acts can be performed with one or another *illocutionary force*, and so constitute *illocutionary acts*, such as assertions, requests, and promises. Statements themselves can be performed with various illocutionary forces: a statement can be asserted or denied, it can be supposed true or supposed false. An *argument* is also a language act, whose premisses and conclusion are themselves illocutionary acts. Attention to language acts leads to an expanded conception of logic, for in addition to considering truth conditions, it proves necessary to consider semantic features due to illocutionary force. These features have an important bearing on whether an argument is satisfactory, and this has not previously been noted.

A system of illocutionary logic is obtained from a standard system by making three changes:

(i) Illocutionary-force indicating expressions, *illocutionary operators*, are added to the artificial language.

(ii) The account of truth conditions is supplemented with an account of *commitment conditions*, which determine which statements a person is committed to accept or reject, once she accepts and rejects some to begin with.

(iii) The deductive system is modified to accommodate illocutionary operators.

An argument conceived as a speech act is not appropriately regarded as valid or not; it is instead *deductively correct* or not. A simple argument is deductively correct if anyone who performs the premiss acts is committed to perform the conclusion act; a complex argument is deductively correct if its components are deductively correct, and anyone who performs the initial premiss acts is committed to perform the final conclusion act. Proofs or deductions in a natural-deduction system can serve as perspicuous representations of deductively correct speech-act arguments.

2. A SIMPLE SYSTEM   I will illustrate a simple system of propositional illocutionary logic. The language *L* contains atomic sentences and compound sentences obtained from them with these connectives: *~, v, &*. (The horseshoe of material implication is a defined symbol.) The atomic and compound sentences are *plain sentences of L*. The plain sentences represent natural-language statements.

The illocutionary operators are the following:

⊢ –the sign of assertion	⊣ –the sign of denial
⌞ –the sign of supposing true	¬ –the sign of supposing false

A plain sentence prefixed with an illocutionary operator is a *completed sentence of L*; there are no other completed sentences. Completed sentences represent illocutionary acts.

An assertion is understood to be an act of producing and coming to accept a statement, or an act of producing and reaffirming (*reflecting*) one's (continued) acceptance of statement. (An assertion in this sense doesn't need an audience, and all such assertions are sincere.) A denial is an act of coming to reject a statement (for being false), or an act reflecting one's rejection of the statement. An argument which begins with assertions and denials can reach a conclusion which is an assertion or denial. An argument which begins with at least one supposition, which remains in force, cannot (correctly) conclude with an assertion or denial. The conclusion will have the force of a supposition, and will be called a supposition.

The semantic account for the language *L* is a two-tier account. The first tier applies to statements apart from illocutionary force. This semantic account gives truth conditions of plain sentences and of the statements that these represent. The first tier of the semantic account presents the *ontology* that the statements encode or represent. The account of truth conditions for plain sentences of *L* is entirely standard: An interpreting function for *L* assigns truth and falsity to the atomic plain sentences, and determines a *truth-value* valuation of the plain sentences in which compound sentences have truth-table values.

The second tier of the semantic account applies to completed sentences and the illocutionary acts they represent. In the case of *L*, it applies to assertions, denials, and suppositions. The second tier of the semantics deals with *rational commitment*. This commitment is distinct from moral or ethical commitment. It is a commitment to perform or not perform some act, or to continue in some state or condition like that of accepting a statement. When we consider the commitment generated by performing acts of assertion, denial, or supposition, this commitment is conditional rather than absolute. A person who accepts a statement will be committed to accept (or continue to accept) some further statement, if the matter comes up and she chooses to think about it–so long as she continues to accept the first statement.

A commitment to perform or not perform an act is always *someone's* commitment. We develop the commitment semantics for an idealized person called the *designated subject*. This subject has beliefs and disbeliefs which are *coherent* in the sense that the beliefs might all be true and the disbeliefs all false.

The second tier of the semantics concerns *epistemology* rather than ontology, but the epistemology must accommodate the ontology. The commitments generated by performing certain illocutionary acts depend on the language user understanding the truth conditions of the statements she asserts, denies, or supposes. We consider the designated subject at some particular moment. There are certain statements which she has considered and accepted, which she remembers and continues to accept. There are similar statements that she has considered and rejected. These *explicit* beliefs and disbeliefs commit her, at that moment, to accept (or continue to accept) further statements and to reject further statements. We use '+' for the value of assertions and denials that she is committed, at that moment, to perform.

A *commitment valuation* is a function which assigns + to some of the assertions and denials in *L*. A commitment valuation *V* is *based on* an interpreting function *f* iff (i) If $V(\vdash A) = +$, then $f(A) = \mathrm{T}$, and (ii) If $V(\dashv A) = +$, then $f(A) = \mathrm{F}$. A commitment valuation is *coherent* iff it is based on an interpreting function.

Let V_0 be a coherent commitment valuation. This can be understood to register the designated subject's explicit beliefs and disbeliefs at a given time. The *commitment valuation* *determined by* V_0 is the function *V* such that (i) *V(⊢A)* = + iff *A* is true for every interpreting function on which V_0 is based, and (ii) *V(⊣A)* = + iff *A* is false for every interpreting function on which V_0 is based. The valuation *V* indicates which assertions and denials the designated subject is committed to perform on the basis of her explicit beliefs and disbeliefs.

A commitment valuation is *acceptable* iff it is determined by a coherent commitment valuation. The following matrices show how acceptable commitment valuations "work": In the matrices, the letter '*b*' stands for *blank*–for those positions in which no value is assigned:

⊢A	*⊢B*	*⊣A*	*⊣B*	*⊢~A*	*⊣~A*	*⊢[A & B]*	*⊣[A & B]*	*⊢[A v B]*	*⊣[A v B]*
+	+	*b*	*b*	*b*	+	+	*b*	+	*b*
+	*b*	*b*	*b*	*b*	+	*b*	*b*	+	*b*
+	*b*	*b*	+	*b*	+	*b*	+	+	*b*
b	+	*b*	*b*	*b*	*b*	*b*	*b*	+	*b*
b	*b*	*b*	*b*	*b*	*b*	*b*	+,*b*	+,*b*	*b*
b	*b*	*b*	+	*b*	*b*	*b*	+	*b*	*b*
b	+	+	*b*	+	*b*	*b*	+	+	*b*
b	*b*	+	*b*	+	*b*	*b*	+	*b*	*b*
b	*b*	+	+	+	*b*	*b*	+	*b*	+

The first row shows the commitments of accepting/asserting both *A, B*, the second shows the commitments of accepting *A* and neither accepting nor rejecting *B*. Etc. In some cases, the values (or non-values) of assertions and denials of simple sentences are not sufficient to determine the values of assertions and denials of compound sentences. For example, if ⊣*A* and ⊣*B* have no value, and *A, B* are irrelevant to one another, then '⊣*[A & B]*' should have no value. But if ⊣*A,* ⊣*~A* have no value, the completed sentence '⊣*[A & ~A]*' will have value +.

3. REASONING It is rational commitment which provides the motive power leading a person from premisses to conclusion in an argument–the arguer needs to recognize that having performed the premiss acts, she is committed to perform the conclusion act before she can be justified in deriving that act (or in using the premiss acts to support the conclusion act).

For the language *L* we employ a natural deduction system *S* which uses tree proofs. Steps in a proof are completed sentences, and the rules take account of illocutionary force. Proofs (deductions) in *S* represent genuine arguments made with the speech acts that the sentences represent. The rule *& Introduction* is the following:

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⊢/⌞A      ⊢/⌞B
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 ⊢/⌞[A & B]
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The expression '⊢/⌞' is used to indicate that the illustration covers both assertions and (positive) suppositions. If both premisses are assertions, so is the conclusion. Otherwise, the conclusion is a supposition.

For an argument understood as a speech act to be *deductively correct*, or valid in an illocutionary sense, we must consider commitment rather than truth conditions. For a simple argument to be deductively correct, performing the premiss acts must commit a person to performing the conclusion act. The following argument is incorrect:

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⌞A      ⌞B
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 ⊢[A & B]
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Even though this argument is truth-preserving, the force of the premisses will not support the force of the conclusion. When one or both premisses are supposed, the conclusion must have the force of a supposition. The following arguments are correct:

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⌞A      ⌞B        ⌞A      ⊢B        ⊢A      ⊢B
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⌞[A & B]          ⌞[A & B]          ⊢[A & B]
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A branch in a tree proof (deduction) begins with an assertion, a denial, or a supposition. An *initial assertion* or *initial denial* is not a hypothesis of the proof. Initial assertions should be of statements known or believed true by the person making the argument, and initial denials should be of statements known to be false or believed to be false. An *initial supposition is* a hypothesis of the proof. An argument/proof establishes that performing the initial illocutionary acts commits a person to performing the conclusion act.

4. SEMANTIC PARALLELS Our system has two semantic levels, and there are corresponding features at each level. Statements (truth-conditionally) *imply* a further statement if there is no interpreting function which satisfies (makes true) the first statements but not the further statement. At the (epistemic) level of illocutionary acts, I prefer to speak of acts *logically requiring* another act, rather than of some kind of implication. To characterize this, I need preliminary definitions:

If *A* is a plain sentence of *L*, and *f* is an interpreting function of *L*, then (i) *f satisfies* ⌞*A* iff *f(A)* = T, and (ii) *f satisfies* ¬*A* iff *f(A)* = F.

If *B* is an assertion or denial of *L*, V_0 is a coherent commitment valuation of *L*, and *V* is the commitment valuation determined by V_0, then V_0 *satisfies B* iff *V(B)* = +.

Completed sentences $A_1, \ldots, A_n$ of *L logically require* completed sentence *B* iff (i) if *B* an assertion or denial, then no coherent commitment valuation satisfies the assertions and denials among $A_1, \ldots, A_n$ and fails to satisfy *B*; (ii) if *B* is a supposition, then there is no interpreting function *f* and commitment valuation V_0 based on *f* such that V_0 satisfies the assertions and denials among $A_1,\ldots, A_n$, *f* satisfies the suppositions among $A_1,\ldots, A_n$, and *f* fails to satisfy *B*.

A set *X* of plain sentences of *L* is *consistent* iff there is an interpreting function of *L* for which every sentence in *X* has value T. A set *X* of completed sentences of *L* is *coherent* iff there is an interpreting function *f* of *L* and a commitment valuation V_0 (of *L*) based on *f* such that V_0 satisfies the assertions and denials in *X* and *f* satisfies the suppositions in *X*.

Although logical requirement corresponds to implication, and coherence corresponds to implication, cases arise where there is a kind of "mismatch" between corresponding features. For typical plain sentences *A*, *B*, we have that ⊢*A* logically requires ⊢*B* iff *A* implies *B*. But a sentence ⊢*A* logically requires ⊢*I believe that A*, even though there is no implication.

And , normally, a set of assertions is coherent just in case the set of the statements being asserted is consistent. However, as G. E. Moore noted, a statement '*A & ~I believe that A*' is consistent, though its assertion is incoherent. These mismatches are not puzzles, though they have puzzled many philosophers, who failed to appreciate that there are two semantic levels.

5. EPISTEMIC MODAL LOGIC People sometimes come to accept statements in a careless or capricious manner. In what follows, I will limit my attention to assertions for which the designated subject is (was) justified in making them. And I employ an epistemic concept of justification such that the person who makes a justified assertion is in a position to realize, upon reflection, that she is justified. There are two kinds of situation I will consider: (1) Those where an assertion has the force of a knowledge claim, (2) Those where the assertion has the force of a claim to justifiably believe/accept the statement involved. The crucial difference here between justified belief and knowledge is that a statement which is justifiably believed may be false, while only a true statement can be known. (But I am *not* defining knowledge when I say this.)

We begin with knowledge. We introduce the modal operator '□,' so that a sentence (statement) □*A* has the sense (for the designated subject) that she is rationally committed by her knowledge to accept *A*. The sentence □*A* can be true without the designated subject realizing that it is true; it can be true without being *explicitly* known by her. But if she accepts/asserts this sentence: ⊢□*A*, then she must either already explicitly know that *A*, or she must be prepared to immediately infer the assertion ⊢*A*.

Our system of modal logic will explore concepts of epistemic necessity and possibility based on knowledge. A statement is epistemically possible at present for someone if it does not conflict with her current knowledge, and is epistemically necessary if she is committed by her current knowledge to accept it. Rational commitment is the basis for epistemic necessity and possibility.

With the simple language *L*, the truth values of plain sentences are entirely determined by the truth values of the (plain) atomic sentences. The commitment values of completed sentences are not dependent on the truth values of their plain components, though we focus on coherent commitment valuations which are based on interpreting functions. Once we enlarge the language *L* with the modal operator '□,' there are plain sentences □*A* whose values are not determined by the values of the atomic sentences. Their truth values depend on commitment values: □*A* is true if ⊢*A* has value +, and is false otherwise.

The semantic account for the modal language is more complicated than I have space to explain, so I will focus on deductive principles and the deductive system for this language. It is our intuitions about deductive principles which guide the development of the semantics.

Since the deductive system for the language *L* is a standard natural-deduction system adapted to accommodate illocutionary operators, we shall focus on modal principles. These principles for reasoning with asserted box sentences are evidently correct:

□ *Introduction*	□ *Elimination*
⊢*A*	⊢□*A*
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⊢□*A*	⊢*A*

That assertions have the status of knowledge claims "cashes out" in the principle □ *Introduction*. If the designated subject asserts *A*, then she is in a position to realize that she knows *A*–that *A* (belongs to or) follows from her current knowledge. (The designated subject, like all of us, can make mistakes about what she knows; but we consider inferences which are correct, given the presumption that she hasn't made a mistake.) And if the designated subject knows that *A* follows from her current knowledge, then she can assert *A* with the force of a knowledge claim.

No other modal rules are needed for reasoning from assertions to assertions. We need to adopt rules for reasoning with positive suppositions. To suppose a non-modal statement is true: ⌞*A*, is not to consider subjunctive or counterfactual circumstances. If the designated subject (who may be us) is committed to accept *A* (⊢*A*), then supposing *A* true is redundant, and if she is committed to reject *A* (⊣*A*), then supposing *A* true produces incoherence in her suppositions. If she is committed to neither accept nor reject *A*, then to suppose *A* true is to consider interpretations of the language in which the statements she is committed to accept are true and those she is committed to reject are false, and in which, in addition, *A* is true. To suppose □*A* is true, when the designated subject is committed neither to accept nor to reject *A*, is to consider an expansion of her explicit beliefs in which *A* is known.

This principle □ *Elimination* is clearly correct:

⌞□*A*

⌞*A*

If we suppose that *A* belongs to (follows from) our current knowledge, then we have also supposed that *A* is true. But, of course, we don't have this: □ *Introduction*

$$\frac{\llcorner A}{\llcorner \Box A}$$

In place of this rejected principle, we have the following two principles for inferring the supposition of a boxed statement:

(S4)

$$\frac{\llcorner \Box A}{\llcorner \Box\Box A}$$

(T)

$$\frac{\llcorner \Box A \qquad \llcorner \Box [A \supset B]}{\llcorner \Box B}$$

The principle *(S4)* is the counterpart for supposition to □ *Introduction* for assertion, while the principle *(T)* captures the idea that knowledge is preserved/enlarged by deductively correct reasoning.

What is interesting about this illocutionary version of the system *S4* is the way that the rules, the inference principles, are divided into those for assertion and those for supposition. Standard systems, by failing to deal with illocutionary acts and illocutionary force, run the two classes of principles together, and then place restrictions on the principle □ *Introduction* so that it is only used with what are, in effect, assertions.

6. JUSTIFIED BELIEF Now we shall let assertions have the force of claims to justifiably believe, and the box be used to say that a statement is either justifiably believed or that its assertion follows from what is justifiably believed or disbelieved. How will the inference principles for knowledge be affected by the switch to justified belief?

The principle □ *Introduction* for assertions presents no problem. Asserting *A* with the force of justified belief commits a person to accept the claim that the assertion of *A* follows from her justified beliefs. But what about □ *Elimination?*

What is known must be true, while what is justifiably believed might not be true. It is reasonable to think that the justification required for a statement to be known is different from or greater than what it takes to authorize belief, although what is known is also believed, and justifiably so. But there might be cases where a person justifiably believes a statement *A*, and also justifiably believes that she knows *A*, when her justification for believing *A* isn't sufficient to yield knowledge. For such cases, the principle □ *Elimination* is not problematic.

However it can also happen that a person justifiably believes a statement A, but realizes that she doesn't know A. Although A is not *known* to be false, which means that $\sim A$ is possible in the "knowledge system" we have just explored, if she justifiably asserts that A follows from her justified beliefs, she is committed to accept A.

But now what about suppositions? The principles *(S4)* and *(T)* are unproblematic for justified belief. If the designated subject is committed by her justified beliefs and disbeliefs to accept A, then she is further committed to accept that she is committed to accept A. And justified belief, like knowledge, is preserved/extended by deductively correct reasoning.

The inference principle for knowledge that becomes problematic when we are dealing with justified belief is this one: $\Box$ *Elimination*

$$\begin{array}{c} \llcorner\Box A \\ \text{------} \\ \llcorner A \end{array}$$

When the designated subject asserts that she is committed to accept A, then, following her commitments, she must assert/accept A. But when she supposes that she is committed to justifiably accept A, she is not at the same time supposing that A is true. For the designated subject to accept a statement is to accept that the world is as the statement says. But to suppose that a statement is believed is only to suppose something about herself. She distinguishes between knowledge and justified belief, and doesn't take herself to be infallible.

Once $\Box$ *Elimination* is dropped for supposition, some other rules/principles are needed to completely capture the reasoning that is correct for the system of justified belief. The following principles seem to do this job:

Suppositional $\Box$ *Introduction*

$$\begin{array}{ccc} & & \{\vdash A_1 \ldots \vdash A_n\} \\ & & \\ \llcorner\Box A_1 \ldots \llcorner\Box A_n & & \vdash B \\ \hline & \llcorner\Box B & \end{array}$$

This rule has the peculiarity that it provides for "hypotheses" which are assertions (those in braces), and which get canceled, or discharged, by an application of the rule. The subproof here is like a proof "on the side." The arguer is "stepping into" a situation in which additional beliefs are justified, and concluding with an assertion with respect to that situation. What takes place in this situation is then "reported" by the conclusion '$\llcorner\Box B$.' This rule makes the rule *(T)* redundant.

◊ *Introduction*

˾□*A*

˾◊*A*

This rule ◊ *Introduction* has the effect of "ruling out" contradictory beliefs (supposed beliefs '˾□*A*' and '˾□~*A*,' or '˾□*[A & ~A]*'). The designated subject's beliefs are coherent.

As it turns out, the logical system for justified belief is different from the system for knowledge, but not so very different. Which, perhaps, is not a very surprising result.

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